MATRICES

Class 12 MATHS

Basics of MATRICES

Tables

	Notebooks	Pens
Radha	15	6
Fauzia	10	2
Simran	13	5

	R	F	S
N.B. →	15	10	13
Pens →	6	2	5

Matrix (आव्यूह) [ ]

$$\begin{bmatrix} 15 & 6 \\ 10 & 2 \\ 13 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 15 & 10 & 13 \\ 6 & 2 & 5 \end{bmatrix}$$

☆ An ordered rectangular array of numbers

(पंक्ति) स्तंभ Columns

$$\begin{array}{c|ccc} & C_1 & C_2 & C_3 \\ \hline \text{Row } (R_1) & 15 & 10 & 13 \\ (R_2) & 6 & 2 & 5 \end{array}$$

~~order~~

Elements (Entries) of matrix.

Order of a matrix (size)/shape.

No. of Rows × No. of Columns

(by)

e.g. = 2×3

$$\begin{array}{c|ccc} & C_1 & C_2 & C_3 \\ \hline R_1 & 15 & 10 & 13 \\ R_2 & 6 & 2 & 5 \end{array}_{2\times 3}$$

$$\begin{array}{c|cc} & C_1 & C_2 \\ \hline R_1 & 15 & 6 \\ R_2 & 10 & 2 \\ R_3 & 13 & 5 \end{array}_{3\times 2}$$

General Representation of Matrices

Construct a matrix 'A' of order $m \times n$.

no. of rows $= m$

—— columns $= n$

$$A = [a_{ij}]_{m\times n}$$

i = Row
j = Column

$$A = \begin{array}{c} \left[\begin{array}{ccccc} \boxed{a_{11}} & \boxed{a_{12}} & \boxed{a_{13}} & \cdots & \boxed{a_{1n}} \\ \boxed{a_{21}} & \boxed{a_{22}} & \boxed{a_{23}} & \cdots & \boxed{a_{2n}} \\ \boxed{a_{31}} & \boxed{a_{32}} & \boxed{a_{33}} & \cdots & \boxed{a_{3n}} \\ \vdots & \vdots & \vdots & & \vdots \\ \boxed{a_{m1}} & \boxed{a_{m2}} & \boxed{a_{m3}} & \cdots & \boxed{a_{mn}} \end{array}\right]_{m\times n} \\ \begin{array}{ccccc} \uparrow & \uparrow & \uparrow & & \uparrow \\ C_1 & C_2 & C_3 & \cdots & C_n \end{array} \end{array} \begin{array}{l} \leftarrow R_1 \\ \leftarrow R_2 \\ \leftarrow R_3 \\ \vdots \\ \leftarrow R_m \ (m^{th} \text{ row}) \end{array}$$

e.g. Construct a matrix A of order 2×3 whose each element $a_{ij} = i^2 - j$.

$$A = \underset{\text{Rows}\ \ \text{Columns}}{[a_{ij}]_{2\times 3}} = \begin{array}{c} \begin{array}{c} R_1 \rightarrow \\ R_2 \rightarrow \end{array} \left[\begin{array}{ccc} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{array}\right]_{2\times 3} \\ \begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ C_1 & C_2 & C_3 \end{array} \end{array}$$

$a_{ij} = i^2 - j$

$$\left.\begin{array}{l} a_{11} = 1^2 - 1 = 0 \\ a_{12} = 1^2 - 2 = -1 \\ a_{13} = 1^2 - 3 = -2 \\ a_{21} = 2^2 - 1 = 3 \\ a_{22} = 2^2 - 2 = 2 \\ a_{23} = 2^2 - 3 = 1 \end{array}\right\}$$

$$A = \begin{bmatrix} 0 & -1 & -2 \\ 3 & 2 & 1 \end{bmatrix}_{2\times 3}$$

Types of Matrices (आव्यूहों के प्रकार)

order = $m \times n$ (row, Column)

① Column Matrix (स्तंभ आव्यूह)

No. of columns = 1

$A = [a_{ij}]_{m \times 1}$

$$A = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}_{3 \times 1}$$

② Row Matrix (पंक्ति आव्यूह)

No. of rows = 1

$A = [a_{ij}]_{1 \times n}$

$$A = \begin{bmatrix} 2 & 3 & -5 & 4 \end{bmatrix}_{1 \times 4}$$

③ Square Matrix (वर्ग आव्यूह)

No. of rows = no. of columns

$(m = n)$

$A = [a_{ij}]_{m \times m}$

$A = [a_{ij}]_{n \times n}$

e.g. $$A = \begin{bmatrix} 2 & 3 & -1 \\ 0 & x & 7 \\ y & -3 & 0 \end{bmatrix}_{3 \times 3}$$

Note: There are two diagonals (विकर्ण) in a sq. matrix.

* Primary (main) (Principal) ✓

Secondary ✗

$$\begin{bmatrix} 2 & 3 & -1 \\ 0 & x & 7 \\ y & -3 & 0 \end{bmatrix}_{3 \times 3}$$

④ Diagonal Matrix (विकर्ण आव्यूह)

Square matrix whose non-diagonal elements $= 0$

e.g.

$$A = \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix}_{2\times 2} \qquad B = \begin{bmatrix} \pi & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 7 \end{bmatrix}_{3\times 3}$$

$$\boxed{A = [a_{ij}]_{m\times m}, \text{ where } a_{ij} = 0, \forall\ i \neq j}$$

⑤ Scalar Matrix (अदिश आव्यूह)

Diagonal matrix whose diagonal elements are same (equal)

$$A = \begin{bmatrix} K & 0 \\ 0 & K \end{bmatrix}_{2\times 2} \qquad B = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{bmatrix}_{3\times 3}$$

Scalar Matrix,

$$A = [a_{ij}]_{m\times m}$$

$$a_{ij} = \begin{cases} K, & i = j \\ 0, & i \neq j \end{cases}$$

⑥ Identity Matrix (तत्समक/इकाई आव्यूह)

Scalar matrix whose diagonal elems $= 1$

$$I_1 = [1]_{1\times 1}$$

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2}$$

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3}$$

⑦ Zero Matrix. (शून्य आव्यूह) (O)

All elements = 0

zero = 0

e.g. $A = \begin{bmatrix} 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Equality of matrices. Two matrices A = B are said to be equal

if (i) order • same

(ii) each corresponding element should also be equal.

$A = \begin{bmatrix} 2 & 3 & 4 \\ -1 & 7 & 0 \end{bmatrix}_{2\times3}$

A = B

$B = \begin{bmatrix} 2 & 3 & 4 \\ -1 & 7 & 0 \end{bmatrix}_{2\times3}$

$B = \begin{bmatrix} 2 & -1 \\ 3 & 7 \\ 4 & 0 \end{bmatrix}_{3\times2}$

MATRICES Exercise 3.1

Q2, Q4 (i), Q6 (iii), Q8, Q9, Q10

Q.2 no. of elements = 24

$A = [a_{ij}]_{m \times n}$ no. of elements = mn

$m \times n$ ↑ order

Possible orders = (24)

1×24 4×6 12×2

2×12 6×4 24×1

3×8 8×3

no. of elements = (13)

1×13 13×1

Q.4, (i) $A = [a_{ij}]_{2 \times 2} \longleftrightarrow a_{ij} = \frac{(i+j)^2}{2}$

Rows Columns

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2 \times 2}$$

$$a_{11} = \frac{(1+1)^2}{2} = \frac{4}{2} = 2$$

$$a_{12} = \frac{(1+2)^2}{2} = \frac{9}{2}$$

$$a_{21} = \frac{(2+1)^2}{2} = \frac{9}{2}$$

$$a_{22} = \frac{(2+2)^2}{2} = \frac{16}{2} = 8$$

$$A = \begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}_{2 \times 2}$$

Q.6 (iii)

$$\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix}_{3\times1} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}_{3\times1}$$

By equality of matrices.

$x+y+z = 9$ —— (1)

$x+z = 5$ —— (2)

$y+z = 7$ —— (3)

eq^n (1) − eq^n (2)

$$\begin{aligned} \cancel{x}+y+\cancel{z} &= 9 \\ -\cancel{x}-\cancel{z} &= -5 \\ \hline y &= 4 \end{aligned}$$

eq^n (1) − eq^n (3)

$$\begin{aligned} x+\cancel{y}+\cancel{z} &= 9 \\ -\cancel{y}-\cancel{z} &= -7 \\ \hline x &= 2 \end{aligned}$$

eq^n (3) : $y+z=7$

$4+z=7$

$\boxed{z=3}$

Q.8 $A = [a_{ij}]_{m\times n}$

Square matrix

$$\begin{bmatrix} - & - & - \\ - & - & - \\ - & - & - \end{bmatrix}_{3\times3}$$

no. of rows = no. of Columns

$m = n$

$\boxed{Q.9}$

$$\begin{bmatrix} 3x+7 & \boxed{5} \\ y+1 & \boxed{2-3x} \end{bmatrix}_{2\times 2} = \begin{bmatrix} 0 & \boxed{y-2} \\ 8 & \boxed{4} \end{bmatrix}_{2\times 2}$$

$3x+7=0 \Rightarrow \boxed{x=-\frac{7}{3}}$

$5=y-2 \Rightarrow y=7$ ✓

$y+1=8 \Rightarrow y=7$ ✓

$2-3x=4 \Rightarrow -2=3x \Rightarrow \boxed{x=-\frac{2}{3}}$

Not possible

$\boxed{Q.10}$

$$A=\begin{bmatrix} \boxed{-} & - & - \\ - & - & - \\ - & - & - \end{bmatrix}_{3\times 3}$$

No. of ways to fill it $= 2$

$\boxed{0 \text{ या } 1}$

Possible entries = 0 या 1

No. of ways to fill all places

$= 2\times 2\times 2\times 2\times 2\times 2\times 2\times 2\times 2 = 2^9$

$= 512$

P&C → multiplication rule of counting

necessary

Addition, Subtraction, Multiplication of Matrices

(+) (−) (×)

Addition & Subtraction of Matrices.

$A = [a_{ij}]_{m\times n}$ $\quad B = [b_{ij}]_{m\times n}$

$A \pm B = [a_{ij}]_{m\times n} \pm [b_{ij}]_{m\times n}$

$= [a_{ij} \pm b_{ij}]_{m\times n}$

corresponding elements

$A+B = B+A$

Commutative

e.g. $A = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}_{2\times 2}$ $\quad B = \begin{bmatrix} 7 & 3 \\ -2 & 5 \end{bmatrix}_{2\times 2}$

$C = \begin{bmatrix} 2 & 3 & 1 \\ 7 & -2 & 0 \end{bmatrix}_{2\times 3}$

$A + B = \begin{bmatrix} 2 & \boxed{-1} \\ 0 & 3 \end{bmatrix}_{2\times 2} + \begin{bmatrix} 7 & \boxed{3} \\ -2 & 5 \end{bmatrix}_{2\times 2}$

$= \begin{bmatrix} 9 & 2 \\ -2 & 8 \end{bmatrix}_{2\times 2}$

$A - B$

$= \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} 7 & 3 \\ -2 & 5 \end{bmatrix}$

$= \begin{bmatrix} -5 & -4 \\ 2 & -2 \end{bmatrix}$

$A + C = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}_{2\times 2} + \begin{bmatrix} 2 & 3 & 1 \\ 7 & -2 & 0 \end{bmatrix}_{2\times 3}$ Not Defined

Multiplication of a Scalar with a Matrix.

$(K \neq 0)$

$A = [a_{ij}]_{m \times n}$

$kA = k[a_{ij}]_{m \times n} = [k a_{ij}]_{m \times n}$

e.g. $A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 7 & -5 \end{bmatrix}_{2 \times 3}$

$kA = K \begin{bmatrix} 2 & 3 & 1 \\ 0 & 7 & -5 \end{bmatrix}_{2 \times 3} = \begin{bmatrix} 2K & 3K & K \\ 0 & 7K & -5K \end{bmatrix}_{2 \times 3}$

$K \neq 0$

$-A = (-1)A = (-1) \cdot \begin{bmatrix} 2 & 3 & 1 \\ 0 & 7 & -5 \end{bmatrix}_{2 \times 3} = \begin{bmatrix} -2 & -3 & -1 \\ 0 & -7 & 5 \end{bmatrix}_{2 \times 3}$

MULTIPLICATION of Two Matrices

$\boxed{\text{कब Possible है ?}}$ $\Downarrow$

$\boxed{\text{कैसे करते है ?}} \rightarrow ?$

$A = [a_{ij}]_{m \times n}$ $\quad B = [b_{ij}]_{n \times k}$

$AB \Longrightarrow$ AB का order $= m \times k$

$\underbrace{m \times n}$ $\underbrace{n \times K}$

$AB \neq BA$

Generally not Commutative

$2 \times 3 \quad 3 \times 4$

B.A

$\Rightarrow$ BA का order $= 2 \times 4$

AB कैसे निकालते है?

e.g.

$A = [a_{ij}]_{2\times 3}$ $\qquad B = [b_{ij}]_{3\times 3}$

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -3 & 4 & 5 \end{bmatrix}_{2\times 3} \qquad B = \begin{bmatrix} 1 & 0 & -2 \\ 7 & 3 & 8 \\ -1 & 3 & 0 \end{bmatrix}_{3\times 3}$$

multiply करते वक्त → पहले वाली matrix (Row)   बाद वाली matrix (Column)

$AB \downarrow 2\times 3$

$$\begin{array}{ccccc} & & C_1 & C_2 & C_3 \\ A.B = \begin{array}{c} R_1 \\ R_2 \end{array} \begin{bmatrix} 2 & -1 & 0 \\ -3 & 4 & 5 \end{bmatrix}_{2\times 3} & & \begin{bmatrix} 1 & 0 & -2 \\ 7 & 3 & 8 \\ -1 & 3 & 0 \end{bmatrix}_{3\times 3} \end{array}$$

$$= \begin{bmatrix} \overbrace{(2\times 1) + (-1\times 7) + 0\times(-1)}^{\text{Due to } R_1 \,\&\, C_1} & \overbrace{2\times 0 + (-1)\times 3 + (0\times -1)}^{R_1 \,\&\, C_2} & (-4)+(-8)+0 \\ (-3) + (28) + (-5) & 0+12+15 & 6+32+0 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -3 & -12 \\ 20 & 27 & 38 \end{bmatrix}_{2\times 3}$$

Note: for multiplication of matrices.

AB ≠ BA Generally

① $AB = BA$

↓ multiplication of ~~Dig~~ Diagonal matrices.

e.g. $A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 0 \\ 0 & 5 \end{bmatrix}$

$$AB = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} -1 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & 15 \end{bmatrix}$$

$$BA = \begin{bmatrix} -1 & 0 \\ 0 & 5 \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & 15 \end{bmatrix}$$

② $AB = O \implies \underbrace{A = 0 \text{ या } B = 0}_{\text{जरूरी नहीं है !!!}}$

↑ zero matrix

e.g. $A = \begin{bmatrix} 0 & -2 \\ 0 & 5 \end{bmatrix}$ $B = \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$

$$AB = \begin{bmatrix} 0 & -2 \\ 0 & 5 \end{bmatrix} \times \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$$

zero matrix

e.g.

AB कैसे निकालते है?

$$A = [a_{ij}]_{2\times 3} \qquad B = [b_{ij}]_{3\times 3}$$

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -3 & 4 & 5 \end{bmatrix}_{2\times 3} \qquad B = \begin{bmatrix} 1 & 0 & -2 \\ 7 & 3 & 8 \\ -1 & 3 & 0 \end{bmatrix}_{3\times 3}$$

multiply करते वक्त → पहले वाली matrix (Row) , बाद वाली matrix (Column)

AB

↓

2×3

$$A.B = \begin{array}{c} R_1 \\ R_2 \end{array} \begin{bmatrix} 2 & -1 & 0 \\ -3 & 4 & 5 \end{bmatrix}_{2\times 3} \begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{bmatrix} 1 & 0 & -2 \\ 7 & 3 & 8 \\ -1 & 3 & 0 \end{bmatrix}_{3\times 3} \end{array}$$

Due to R_1 & C_1 ; R_1 & C_2

$$= \begin{bmatrix} (2\times 1) + (-1\times 7) + 0\times(-1) & 2\times 0 + (-1)\times 3 + (0\times -1) & (-4)+(-8)+0 \\ (-3) + (28) + (-5) & 0+12+15 & 6+32+0 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -3 & -12 \\ 20 & 27 & 38 \end{bmatrix}_{2\times 3}$$

Exercise 3.2 $\boxed{Q6,\ Q7\ (ii),\ Q13,\ Q17}$

Class 12 - Maths

$\boxed{+,\ -,\ \times}$

$\boxed{Q.6}$ Simplify $\cos\theta \cdot \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$

$$= \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ -\sin\theta\cdot\cos\theta & \cos^2\theta \end{bmatrix}_{2\times2} + \begin{bmatrix} \sin^2\theta & -\sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}_{2\times2}$$

$$= \begin{bmatrix} \underbrace{\cos^2\theta + \sin^2\theta}_{1} & \cos\theta\cancel{\sin\theta} - \cancel{\sin\theta}\cos\theta \\ 0 & \underbrace{\cos^2\theta + \sin^2\theta}_{1} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times2} = I = I_2 = I_{2\times2}$$

Identity / unit matrix.

$\boxed{Q.7}$ (ii) $2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ —(1) $\quad 3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$ —(2)

By Elimination

eqⁿ (1) × 3 $\quad 6X + 9Y = 3\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 9 \\ 12 & 0 \end{bmatrix}$

eqⁿ (2) × 2 $\quad 6X + 4Y = 2\begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ -2 & 10 \end{bmatrix}$

$\quad - \quad\ - \quad\quad\quad\quad\quad\quad\quad\quad -$

$$5Y = \begin{bmatrix} 6 & 9 \\ 12 & 0 \end{bmatrix} - \begin{bmatrix} 4 & -4 \\ -2 & 10 \end{bmatrix}$$

$$\Rightarrow 5Y = \begin{bmatrix} 2 & 13 \\ 14 & -10 \end{bmatrix} \Rightarrow Y = \frac{1}{5}\begin{bmatrix} 2 & 13 \\ 14 & -10 \end{bmatrix}$$

$$\Rightarrow Y = \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix} \checkmark$$

$$2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - \underbrace{3\begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}}_{3Y}$$

$$\Rightarrow 2X = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - \begin{bmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & -6 \end{bmatrix}$$

$$\Rightarrow \underline{2X = \begin{bmatrix} \frac{4}{5} & -\frac{24}{5} \\ -\frac{22}{5} & 6 \end{bmatrix}} \Rightarrow X = \begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix} \checkmark$$

Q.13 If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$; show that $F(x).F(y) = F(x+y)$

To Prove $F(x).F(y) = F(x+y)$

$$\Rightarrow \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$LHS = F(X).F(y) \rightarrow \cdot \downarrow$

$$= \underset{3\times 3}{\begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}} \cdot \underset{3\times 3}{\begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

$$= \begin{bmatrix} \cos x.\cos y - \sin x \sin y + 0 & -\cos x \sin y - \sin x \cos y + 0 & 0+0+0 \\ \sin x \cos y + \cos x \sin y + 0 & -\sin x \sin y + \cos x \cos y + 0 & 0 \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix} = F(x+y)$$

$= RHS.$

$\boxed{Q.17}$ If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A^2 = kA - 2I$. Find 'k'.

$$A^2 = A \times A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \times \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$$

$$A^2 = kA - 2I$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = k\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = k\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = k\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$\Rightarrow 1.\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = k\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix}$$

$$3 = 3k \Rightarrow k = 1$$

Exercise - 3.2 $\boxed{Q.18}$ ☆

$\boxed{\text{Class-12 Maths}}$

If $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the Identity matrix of order 2, show that $(I + A) = (I - A)\cdot\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

✓ $\sin\alpha = \dfrac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$ ✓

✓ $\cos\alpha = \dfrac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$ ✓

$\sin 2\theta = 2\sin\theta\cos\theta$

$\sin 2\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$

$\cos 2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

multiply $\quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

To Prove: $(I + A) = (I - A)\cdot\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix} = \left\{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}\right\}\cdot$

$\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix} \times \begin{bmatrix} \frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} & -\frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} \\ \frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} & \frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} \end{bmatrix}$$

$\overrightarrow{[\ ]}\cdot[\ ]\downarrow$

$$= \begin{bmatrix} \frac{1-\tan^2\frac{\alpha}{2}+2\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} & \frac{-2\tan\frac{\alpha}{2}+\tan\frac{\alpha}{2}-\tan^3\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} \\ \frac{-\tan\frac{\alpha}{2}+\tan^3\frac{\alpha}{2}+2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} & \frac{2\tan^2\frac{\alpha}{2}+1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} \end{bmatrix}$$

$$= \begin{bmatrix} \cancel{\frac{1+\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}} & -\frac{\tan\frac{\alpha}{2}\cancel{(1+\tan^2\frac{\alpha}{2})}}{\cancel{(1+\tan^2\frac{\alpha}{2})}} \\ \frac{\tan\frac{\alpha}{2}\cancel{(1+\tan^2\frac{\alpha}{2})}}{\cancel{(1+\tan^2\frac{\alpha}{2})}} & \cancel{\frac{1+\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix} = LHS$$

Exercise 3.2

Class - 12

Q 19, Q 20, Q 21, Q 22

Q.19

₹ 30,000

₹ x ₹ $(30,000 - x)$

5% 7%

Matrix (Money) $M = \begin{bmatrix} x & 30000-x \end{bmatrix}_{1\times 2}$

Matrix (Rate) $R = \begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix}_{2\times 1}$ ← 5% ← 7%

Matrix = Interest = $T = M.R$

1×2, 2×1, 1×1

$$T = \begin{bmatrix} x & 30000-x \end{bmatrix} . \begin{bmatrix} 5/100 \\ 7/100 \end{bmatrix}$$

$$T = \begin{bmatrix} x \times \frac{5}{100} + (30000-x) \times \frac{7}{100} \end{bmatrix}_{1\times 1} =$$

Part (i) $\begin{bmatrix} \frac{5x}{100} + \frac{210000-7x}{100} \end{bmatrix} = \begin{bmatrix} 1800 \end{bmatrix}$

$$\therefore \frac{5x}{100} + \frac{210000-7x}{100} = 1800$$

$$\Rightarrow 5x + 2,10,000 - 7x = 1,80,000$$

$$\Rightarrow 30,000 = 2x$$

$$\Rightarrow x = 15000$$

$$30000 \begin{cases} x = 15000 \\ 30000 - x = 15000 \end{cases}$$

Part (ii)

$$\left[\frac{5x}{100} + \frac{210000-7x}{100}\right]_{1\times 1} = [2000]_{1\times 1}$$

$$\Rightarrow 5x + 210000 - 7x = 200000$$

$$\Rightarrow 10000 = 2x$$

$$\Rightarrow x = 5000$$

$$30000 \begin{cases} x = 5000 & 5\% \\ 30000 - x = 25000 & 7\% \end{cases}$$

Q. 20 Matrix = Books = B = $\begin{array}{ccc} C & P & E \\ \left[120\right. & 96 & \left.120\right]_{1\times3} \end{array}$

Matrix = Price = P = $\begin{bmatrix} 80 \\ 60 \\ 40 \end{bmatrix}$ $\begin{array}{l} \leftarrow C \\ \leftarrow P \\ \leftarrow E \end{array}$

All amount

$\underset{(1\times1)}{A} = \begin{bmatrix} 120 & 96 & 120 \end{bmatrix}_{1\times3} \cdot \begin{bmatrix} 80 \\ 60 \\ 40 \end{bmatrix}_{3\times1}$

no. of Books

Price.

$\underset{(1\times1)}{A} = \begin{bmatrix} (9600) + (5760) + (4800) \end{bmatrix}_{1\times1}$

$A = \begin{bmatrix} 20160 \end{bmatrix}_{1\times1}$

matrix for all amount. = ₹ 20,160

$$X \rightarrow (2 \times n) \quad Y \rightarrow (3 \times K) \quad Z \rightarrow (2 \times p) \quad W \rightarrow (n \times 3) \quad P \rightarrow (p \times K)$$

$A \pm B$ → order same

Q. 21 n, K, p

"$\overbrace{P}^{p \times K}.Y + \overbrace{W}^{n \times 3}.Y$" will be defined

$\underbrace{p \times K \quad 3 \times K}_{K=3}$ $\quad$ $\underbrace{n \times 3 \; . \; 3 \times K}_{3}$

order of $P.Y$ = order of $W.Y$

$p \times K \quad n \times 3$

$p = n$, $K = 3$ $\quad$ Option (A)

Q.22 If $n = p$, then order of "$7X - 5Z$" = ?

'$7X - 5Z$' = "Ansuer Matrix"

$(2 \times n) \quad (2 \times p)$

$2 \times n \quad 2 \times n$

या

$2 \times p \quad 2 \times p$

$2 \times n$

या

$2 \times p$

Option (B)

MATRICES Transpose, Symmetric Matrices

Skew Symmetric Matrices

Transpose (A^T / A')

let $A = [a_{ij}]_{m \times n}$ then it's transpose

A' or $A^T = [a_{ji}]_{n \times m}$ (Interchange rows with columns)

e.g. $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 6 & 7 \end{bmatrix}_{3 \times 2}$ $A^T = A' = \begin{bmatrix} 2 & 4 & 6 \\ 3 & 5 & 7 \end{bmatrix}_{2 \times 3}$

$B = \begin{bmatrix} 0 & 2 \\ -1 & 5 \end{bmatrix}_{2 \times 2}$ $B' = B^T = \begin{bmatrix} 0 & -1 \\ 2 & 5 \end{bmatrix}_{2 \times 2}$

Properties of Transpose.

(i) $(A^T)^T = A$ (ii) $(kA)^T = kA^T$ ($k \in R$, $k \neq 0$)

(iii) $(A \pm B)^T = A^T \pm B^T$ $(A+B+C)^T = A^T + B^T + C^T$

* (iv) $(AB)^T = B^T . A^T$ Reversal Law.

$(AB)^T \neq A^T . B^T$ $AB \neq BA$ Generally

$(ABC)^T = C^T . B^T . A^T$ Generally

Symmetric Matrices

(Square matrix)

$$A^T = A$$

$$[a_{ji}]_{n\times n} = [a_{ij}]_{n\times n}$$

$$a_{ji} = a_{ij}$$

e.g. $A = \begin{bmatrix} 2 & 0 & -7 \\ 0 & 3 & 4 \\ -7 & 4 & -1 \end{bmatrix}_{3\times 3}$

$$A^T = \begin{bmatrix} 2 & 0 & -7 \\ 0 & 3 & 4 \\ -7 & 4 & -1 \end{bmatrix}_{3\times 3}$$

$\boxed{A^T = A}$ Sym. matrix.

Skew Symmetric Matrices

(Square matrix)

$$A^T = -A$$

$$[a_{ji}]_{n\times n} = -[a_{ij}]_{n\times n}$$

$$a_{ji} = -a_{ij}$$

For main Diagonal elements $(i=j)$

$$a_{ii} = -a_{ii}$$

$\Rightarrow 2a_{ii} = 0$

$\Rightarrow \boxed{a_{ii} = 0}$ ← Diagonal elements.

main Diagonal $(i=j)$

e.g. $B = \begin{bmatrix} 0 & 2 & 4 \\ -2 & 0 & -3 \\ -4 & 3 & 0 \end{bmatrix}_{3\times 3}$

$$B^T = \begin{bmatrix} 0 & -2 & -4 \\ 2 & 0 & 3 \\ 4 & -3 & 0 \end{bmatrix}$$

$$= -\begin{bmatrix} 0 & 2 & 4 \\ -2 & 0 & -3 \\ -4 & 3 & 0 \end{bmatrix}$$

$$B^T = -B$$

Theorem ① A is any square matrix

(i) $A + A^T$ is always symmetric.

(ii) $A - A^T$ is always skew symmetric.

Proof: let $X = A + A^T$

$X^T = (A + A^T)^T = A^T + (A^T)^T = A^T + A = X$

$X \to$ sym.

$A + A^T \to$ sym.

Let $Y = A - A^T$

$(Y)^T = (A - A^T)^T = A^T - (A^T)^T = A^T - A = -(A - A^T)$

$Y^T = -Y$

$\therefore$ $Y \to$ skew sym.

$A - A^T \to$ skew sym.

Theorem ② 'A' $\to$ any sq. matrix.

$A =$ (sym.) + (skew sym.)

$$\boxed{A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)}$$

e.g. $O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (zero matrix)

$\boxed{\begin{matrix} O^T = O \\ O^T = -O \end{matrix}}$

↓

Both → Sym.
→ Skew sym.

e.g. $A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & 10 \end{bmatrix}$ → Sym ✗
→ Skew sym ✗

$A^T = \begin{bmatrix} 2 & 5 & 8 \\ 3 & 6 & 9 \\ 4 & 7 & 10 \end{bmatrix}$

Sym. part $= \frac{1}{2}(A + A^T)$

$= \frac{1}{2}\left(\begin{bmatrix} 4 & 8 & 12 \\ 8 & 12 & 16 \\ 12 & 16 & 20 \end{bmatrix}\right)$

$= \begin{bmatrix} 2 & 4 & 6 \\ 4 & 6 & 8 \\ 6 & 8 & 10 \end{bmatrix}$ ✓

Skew sym. Part $= \frac{1}{2}(A - A^T)$

$= \frac{1}{2}\left(\begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & 10 \end{bmatrix} - \begin{bmatrix} 2 & 5 & 8 \\ 3 & 6 & 9 \\ 4 & 7 & 10 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 0 & -2 & -4 \\ 2 & 0 & -2 \\ 4 & 2 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$

$A = \underbrace{\frac{1}{2}(A + A^T)}_{\text{Sym.}} + \underbrace{\frac{1}{2}(A - A^T)}_{\text{Skew}} = \begin{bmatrix} 2 & 4 & 6 \\ 4 & 6 & 8 \\ 6 & 8 & 10 \end{bmatrix} + \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$

MATRICES Exercise 3.3 $\boxed{Q5, Q6, Q10, Q11, Q12}$

$\boxed{Q.5}$ (i) $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}_{3\times 1}$ $B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}_{1\times 3}$

$(AB)' = \cancel{A'B'}\ B'A'$

$$AB = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} -1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}$$

$$LHS = (AB)' = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}' = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$$

$$RHS = B'A' = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}' \cdot \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}'$$

$$= \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix} = LHS$$

$(AB)' = B'A'$

(6) (i) $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}_{2\times 2}$

<u>To Prove</u>

$A'.A = I$

$2\times 2 \quad 2\times 2 \quad 2\times 2$

$I =$ Identity matrix (2×2)

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = RHS.$$

$$LHS = A'A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}' . \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} . \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$\longrightarrow . \downarrow$

$$= \begin{bmatrix} \overset{1}{\cos^2\alpha + \sin^2\alpha} & \overset{0}{\cancel{\cos\alpha\sin\alpha} - \cancel{\sin\alpha\cos\alpha}} \\ \underset{0}{\sin\alpha\cancel{\cos\alpha} - \cancel{\cos\alpha}\sin\alpha} & \underset{1}{\sin^2\alpha + \cos^2\alpha} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2} = I = RHS.$$

$\boxed{Q.10}$ (iii) $A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$

$A = (\text{a } \underbrace{\text{symmetric matrix}}) + (\text{a skew sym. matrix})$

$A = \frac{1}{2}A + \frac{1}{2}A$

$$A = \underbrace{\frac{1}{2}(A+A')}_{\text{Sym.}} + \underbrace{\frac{1}{2}(A-A')}_{\text{Skew sym.}}$$

$X^T = X$ $\quad$ $X^T = -X$

$A' = \begin{pmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{pmatrix}$

$RHS = \frac{1}{2}(A+A') + \frac{1}{2}(A-A')$

$= \frac{1}{2}\left\{\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}\right\}$

$+ \frac{1}{2}\left\{\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}\right\}$

$A = \frac{1}{2}\left\{\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}\right\} + \frac{1}{2}\begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix}$

(11) $A \rightarrow A^T = A$
$B \rightarrow B^T = B$ (Sym.)

$(A \pm B)^T = A^T \pm B^T$

$(AB)^T = B^T A^T$

$X = AB - BA$

$X^T = (AB - BA)^T = (AB)^T - (BA)^T$

$= B^T A^T - A^T . B^T$

$= BA - AB$

$= -(AB - BA)$

$X^T = -(X)$

'X' is skew sym.

$\Rightarrow$ 'AB-BA' is skew sym.

Option (A)

Q.12 $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}_{2\times 2}$

$A + A' = I$

$\Rightarrow \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \boxed{\cos\alpha} \end{bmatrix}_{2\times 2} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \boxed{\cos\alpha} \end{bmatrix}_{2\times 2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2}$

$\Rightarrow \begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\alpha = ?$

Comparison. $2\cos\alpha = 1$

$\cos\alpha = \frac{1}{2}$

Option - B

$\alpha = 60^\circ = \frac{\pi}{3}$

Elementary Operations (प्राथमिक संक्रियाएं) | Invertible Matrices (व्युत्क्रमणीय आव्यूह) | Inverse of a matrix by Elementary Operations

Invertible Matrices. let 'A' → order 'm'

(Square matrix) (order $m \times m$)

let matrix 'B' exists for 'A' such that

$$\underset{m\times m}{A}\underset{m\times m}{B} = BA = \underset{\text{Identity matrix } (m\times m)}{I}$$

Generally $AB \neq BA$

then 'A' is called Invertible Matrix.

A का inverse $= A^{-1} = B$

B का inverse matrix $= B^{-1} = A$

Note. Rectangular matrices $[\ \ ]_{2\times3}$ $[\ ]_{3\times1}$
↓
These are not invertible.

Note: ① A के A^{-1} (B) unique होता है

② $(AB)^{-1} = B^{-1}.A^{-1}$ (Reversal Law)

~~$(AB)^{-1} = A^{-1}B^{-1}$~~ $(AB)^T = B^T.A^T$

e.g. $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}_{2\times 2}$ $B = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}_{2\times 2}$

$AB = BA = I \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2}$

AB

$= \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 4-3 & -\cancel{6}+\cancel{6} \\ \cancel{2}-\cancel{2} & -3+4 \end{bmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$

$BA = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$AB = BA = I.$

$\therefore$ A का Inverse $= B$

B का inverse $= A$

Elementary Operations on a Matrix

Elementary Row operation

Elementary Column operation.

(I) $R_i \leftrightarrow R_j$ $C_i \leftrightarrow C_j$

$$\begin{array}{c} R_1 \leftarrow \\ R_2 \leftarrow \\ R_3 \leftarrow \end{array} \begin{bmatrix} 2 & 4 & 3 \\ 7 & 8 & 0 \\ -1 & -2 & -3 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 2 & 4 & 3 \\ -1 & -2 & -3 \\ 7 & 8 & 0 \end{bmatrix}$$

How to find inverse of 'A'

$A^{-1} = A$? let inverse of $A = B = A^{-1}$

$B = ?$

Given: 'A' square matrix

$AB = BA = I$

$A = A$

$A = I.A$ ($\because IA = AI = A$)

$I \rightarrow$ multiplicative identity for matrices.

Identity matrix

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times2}$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times3}$

$2\times1 = 2$
$3\times1 = 3$

A से I लाना

Elementary Row Operation.

$I = BA$

$A \times A^{-1} = I$

$A^{-1} =$ A का inverse $= B$

using Ele. Column ~~col~~ op.

$A = A.I$

$I = AB$

A^{-1}

(ii) $R_i \to kR_i$ $(k \neq 0)$ $C_i \to kC_i$ $(k \neq 0)$

$$\begin{array}{c} \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}_{2\times 2} \\ \begin{array}{cc} C_1 & C_2 \end{array} \end{array} \xrightarrow{C_2 \to -3C_2} \begin{bmatrix} 2 & -9 \\ 4 & -3 \end{bmatrix}$$

(iii) $R_i \to \underbrace{R_i \pm kR_j}$ $C_i \to C_i \pm kC_j$

$$\begin{array}{c} R_1 \to \\ \\ R_3 \to \end{array} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \xrightarrow{R_3 \to \underbrace{R_3 - 7R_1}} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 0 & -6 & 12 \end{bmatrix}$$

$\to 7 - 7\times(1) = 0$

$\to 8 - 7\times 2 = -6$

$\to 9 - 7\times 3 = 12$

How to find Inverse (A^{-1}) of a matrix (A) using Elementary transformations (operations)

Note:

Ele. Row op. (only on Left matrix)

Ele. Column op. (only on Right matrix)

Equation (समीकरण)

$A.B = C.D.E$

$X = AB$

$A.B = C.D.E$

$X = AB$

e.g. Find inverse of $\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}_{2\times 2}$.

(Using elementary row operations)

Ans. $A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$ left

$A = I \cdot A$ left

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2}$

$\Rightarrow \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot A$ R_2

Final Target

$I = B \cdot A$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = [\quad] \cdot A$

A^{-1}

Elementary Row operation

$R_2 \rightarrow R_2 - 2R_1$

$2 \rightarrow 2 - 2\times 1 = 0$

$7 \rightarrow 7 - 2\times 3 = 7 - 6 = 1$

$0 \rightarrow 0 - 2\times 1 = -2$

$1 \rightarrow 1 - 2\times 0 = 1 - 0 = 1$

$R_i \leftrightarrow R_j$

$R_i \rightarrow kR_i$

$R_i \rightarrow R_i \pm kR_j$

$k \neq 0$

$\Rightarrow \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \cdot A$

$R_1 \rightarrow R_1 - 3R_2$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix} \cdot A$

$\Rightarrow I = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix} \cdot A$

A^{-1}

$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$ का inverse $= \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$

Exercise 3.4 Q.11 Q12

2×2 matrix का inverse

[Using Elementary Row Transformations]

$A = IA$

Q.11 $A = \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}_{2\times 2}$

I = Identity matrix

= 2×2

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A = I.A$

Ele. Row op. left side

① $R_i \leftrightarrow R_j$

② $R_i \rightarrow kR_i \ (k \neq 0)$

③ $R_i \rightarrow R_i \pm kR_j \ (k \neq 0)$

$AB = BA = I$

B = A का inverse

$B = A^{-1}$

Target set

$I = BA$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = B\begin{bmatrix} & \\ & \end{bmatrix}.A$

$A = I\,A$

$A = IA$

$$\begin{array}{c} R_1 \rightarrow \\ R_2 \rightarrow \end{array} \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.A$$

$R_2 \leftrightarrow R_1$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 2 & -6 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.A$$

$R_2 \to R_2 - 2R_1$

$2 - 2\times 1$
$-6 - 2\times(-2) = -6+4$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix} \cdot A$$

R_2

$$\boxed{R_2 \to \frac{R_2}{-2}}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{2} & 1 \end{bmatrix} \cdot A$$

$R_1 \to R_1 + 2R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix} A$$

$$\Rightarrow I = \begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix} \cdot A$$

$$A = \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$$

$$\boxed{I = B \cdot A}$$

$$B = A^{-1} = \begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix}$$

$\boxed{Q.12}$

$$A = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$$

(Elementary Row operations)

$A = I A$

$$\Rightarrow \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} . A$$

Target

$I = BA$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = [\ \ ] . A$$

$R_1 \to R_1 + 3R_2$

$6 \Rightarrow 6 + 3(-2)$

$6 - 6 = 0$

$-3 \to -3 + 3(1)$

$= 0$

$1 \to 1 + 3(0)$

$= 1$

$0 \to 0 + 3(1)$

$= 3$

$$\Rightarrow \begin{bmatrix} 0 & 0 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} . A$$

1. Complete Row/Column Elements → 0

Target

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = [\ \ ] . A$$

$\therefore A = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$ is not invertible.

3x3 Matrix का Inverse (A^{-1}) = ? How
(using Elementary Row operations)

Exercise 3.4 Q.17

① $R_i \leftrightarrow R_j$
② $R_i \rightarrow kR_i$
③ $R_i \rightarrow R_i \pm kR_j$

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}_{3\times3}$$

Ele. Row. opn.
left matrix.

$$\boxed{I = AB = BA}$$ (A^{-1})

$$\boxed{I = BA}$$

Target

$A = I \cdot A$

I = identity M.

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

$R_1 \rightarrow \frac{R_1}{2}$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

$R_2 \rightarrow R_2 - 5R_1$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

$R_3 \to R_3 - R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ \frac{5}{2} & -1 & 1 \end{bmatrix} \cdot A$$

R_3

$R_3 \to 2R_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ 5 & -2 & 2 \end{bmatrix} \cdot A$$

$-\frac{5}{2} - \frac{5}{2}(5) = \frac{-30}{2}$

$1 + \cancel{\frac{5}{2}(2)} = 6$

$R_2 \to R_2 - \frac{5}{2} R_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} \cdot A$$

$R_1 \to R_1 + \frac{R_3}{2}$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} \cdot A$$

$$I = \underbrace{\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}}_{A^{-1}} \cdot A$$

Miscellaneous Exercise on Chapter - 3

① $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}_{2\times 2}$ Prove: $P(n)$: $\boxed{(aI + bA)^n = a^n.I + na^{n-1}b.A}$ $n \in N$

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$= O$ (ओ)

(zero matrix)

$O.B = O$

$I.B = B$

$B.I = B$

I^2

$I.I = I$

PMI

Principle of Mathematical Induction

Ⓘ $n=1$ के लिए Prove.

Ⓘⓘ $n=K$ (माना)

$n=K+1$ के लिए Prove.

Step-I: $P(1)$ is true??

$n=1$

$P(n): (aI+bA)^n = a^n I + na^{n-1}.b.A$

$P(1): aI + bA = aI + bA$

$\therefore$ $P(1)$ is true.

Step-Ⓘⓘ: let (माना) $P(K)$ is true

$\hookrightarrow (aI+bA)^K = a^K I + Ka^{K-1}.bA$ —— ①

Now we have to prove that $P(K+1)$ is true.

$$\boxed{(aI+bA)^{K+1} = a^{K+1}.I + (K+1).a^K.bA}$$

$$\text{LHS} = (aI + bA)^{K+1}$$

$$= \boxed{(aI+bA)^K} \cdot (aI+bA)$$

$\downarrow$ by eq^n ①

$$= \left(a^K I + K a^{K-1} bA\right) \cdot (aI + bA)$$

$$= a^{K+1} \cdot I^2 + a^K b \cdot IA + K a^K b \cdot AI + \underbrace{K a^{K-1} b^2 \cdot A^2}$$

$I^2 \to I$, $IA \to A$, $AI \to A$, $A^2 \to 0$ (zero matrix)

$A^2 = 0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$5 \cdot A^2 = 5 \cdot 0 = 5 \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$= a^{K+1} \cdot I + a^K b \cdot A + K a^K b \cdot A + 0$$

zero matrix

$$= a^{K+1} \cdot I + (1+K) a^K b A = \text{RHS}$$

$\therefore$ $P(K+1)$ is also true.

$\therefore$ $P(n): (aI + bA)^n = a^n I + n a^{n-1} bA$

is true. (by using PMI)

Miscellaneous Exercise on Chapter 3 — Q4, Q5

Q.4 $\left.\begin{array}{l} A^T = A \\ B^T = B \end{array}\right\} \leftarrow$ Sym. matrices

To Prove $AB - BA \rightarrow$ Skew Sym.

$$(AB-BA)^T = -(AB-BA)$$

$$(XY)^T = Y^T . X^T$$

$$(AB-BA)^T = (AB)^T - (BA)^T$$

$$= B^T . A^T - A^T . B^T$$

$$= B.A - A.B$$

$$\boxed{(AB-BA)^T = -(AB-BA)}$$

$\therefore AB-BA$ is Skew Sym.

Q.5 $B'AB$

$$(X^T)^T = X$$

$$(B'.A.B)' = B'A'(B')'$$

$$\boxed{(B'AB)' = B'A'B} \longrightarrow (1)$$

Part (I) let 'A' is Sym. $\Longrightarrow A^T = A = A'$

by eq^n (1): $(B'AB)' = B'AB$ $\therefore$ Symmetric

Part (II) let A is Skew sym. $\Rightarrow A^T = -A = A'$

By eq^n (1): $(B'AB)' = B'(-A).B = -B'AB \longrightarrow$ Skew Sym.

Q.9

$$\begin{bmatrix} x & -5 & -1 \end{bmatrix}_{1\times 3} \cdot \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}_{3\times 3} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$$

zero matrix

$$\Rightarrow \begin{bmatrix} x-2 & -10 & 2x-8 \end{bmatrix}_{1\times 3} \cdot \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix}_{3\times 1} = O$$

$x = ?$

(ओ)

zero matrix

$$\Rightarrow \begin{bmatrix} (x-2).x - 40 + 2x - 8 \end{bmatrix}_{1\times 1} = \begin{bmatrix} 0 \end{bmatrix}_{1\times 1}$$

Zero

By Comparison,

$$\Rightarrow x^2 - \cancel{2x} - 40 + \cancel{2x} - 8 = 0$$

$$\Rightarrow x^2 - 48 = 0$$

$48 = 16 \times 3$

$$\Rightarrow x^2 = 48$$

$$\Rightarrow x = \pm\sqrt{48}$$

$$\Rightarrow \boxed{x = \pm 4\sqrt{3}}$$

Q.10 Miscellaneous Exercise on chapter 3

Market	x	y	z
I	10000	2000	18000
II	6000	20000	8000

Matrix = $\underset{2\times 3}{M} = \begin{array}{c} \begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix} \begin{array}{l} \to \text{I} \\ \to \text{II} \end{array} \\ \begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ x & y & z \end{array} \end{array}$

market

Matrix = $\underset{3\times 1}{S} = \begin{bmatrix} 2.5 \\ 1.5 \\ 1 \end{bmatrix} \begin{array}{l} \leftarrow x \\ \leftarrow y \\ \leftarrow z \end{array}$

Selling price (₹)

Revenue (matrix) $= \underset{2\times 1}{R} = \underset{2\times 3}{M} \times \underset{3\times 1}{S} = \begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix} \cdot \begin{bmatrix} 2.5 \\ 1.5 \\ 1 \end{bmatrix}$

$$= \begin{bmatrix} 25000 + 3000 + 18000 \\ 15000 + 30000 + 8000 \end{bmatrix}_{2\times 1}$$

$$= \begin{bmatrix} 46000 \\ 53000 \end{bmatrix}_{2\times 1} \begin{array}{l} \to \text{market I's Revenue} \\ \to \text{market II's revenue} \end{array}$$

SP

(II)

$$\text{matrix} = C = \begin{bmatrix} 2 \\ 1 \\ 0.5 \end{bmatrix}_{3\times1}$$

Cost price (₹) (CP)

$$\text{Total cost} = T = M \times C$$

T: 2×1, M: 2×3, C: 3×1

$$= \begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix} \times \begin{bmatrix} 2 \\ 1 \\ 0.5 \end{bmatrix}$$

$$T = \begin{bmatrix} 20000 + 2000 + 9000 \\ 12000 + 20000 + 4000 \end{bmatrix}_{2\times1}$$

$$T = \begin{bmatrix} 31000 \\ 36000 \end{bmatrix}_{2\times1} \begin{matrix} \rightarrow \text{I} \\ \rightarrow \text{II} \end{matrix} \qquad R = \begin{bmatrix} 46000 \\ 53000 \end{bmatrix} \begin{matrix} \rightarrow \text{I} \\ \rightarrow \text{II} \end{matrix}$$

total cost (CP) Revenue (S.P.)

Gross profit $= G = R - T$

matrix,

$$= \begin{bmatrix} 46000 \\ 53000 \end{bmatrix} - \begin{bmatrix} 31000 \\ 36000 \end{bmatrix}$$

$$= \begin{bmatrix} 15000 \\ 17000 \end{bmatrix} \begin{matrix} \leftarrow \text{I} \\ \leftarrow \text{II} \end{matrix}$$

Miscellaneous Exercise on Chapter-3

Q11, Q12

Q.11

$$\underset{\text{Matrix}}{X} \cdot \underbrace{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times 3}}_{A} = \underbrace{\begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}_{2\times 3}}_{B}$$

$$X \cdot A = B$$

$$\boxed{2} \times \boxed{2} \quad 2\times 3 \qquad 2\times 3$$

X का order $= 2\times 2$

Let $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times 2}$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

multiply $\rightarrow$ solve $\rightarrow$ Compare

equations

a, b, c, d $\leftarrow$ solve $\leftarrow$

Q.12 $AB = BA$ (Given)

To Prove (I) $AB^n = B^n A$ $n \in N$ — Statement $P(n)$

(II) $(AB)^n = A^n B^n$

(I)

Let

$P(n): AB^n = B^n A$

1^{st} step:

$n = 1$

$P(1): AB = BA$

True.

P(1) is true.

Principle of mathematical Induction: (PMI)

(I) $n=1$ के लिए Prove.

(II) $n = K$ (माना) $P(K)$ ✓ — (1)

$n = K+1$ के लिए prove.

Target

$P(K+1): AB^{K+1} = B^{K+1}.A$

2^{nd} step: Let $P(K)$ is true

$P(K): \boxed{AB^K = B^K A}$ —— (1)

Now we have to prove that $P(K+1)$ is also true (i.e. $AB^{K+1} = B^{K+1}.A$)

$LHS = AB^{K+1}$

$= A.B^K . B$

By eq^n (1)

$= B^K.A.B$ $\quad (\because AB = BA$ Given$)$

$= B^K . BA$

$= B^{K+1}.A = RHS.$

$\therefore$ P(K+1) is true

$\therefore P(n)$:

$AB^n = B^n A$

is true.

H.P

(II) To Prove $P(n): (AB)^n = A^n . B^n$ $n \in N$

1^{st} step.

$P(1)$ is true.

~~$P(1) = (AB)^n$~~ = $P(1): (AB)^1 = A^1 . B^1$

$\Rightarrow AB = AB$

$\therefore P(1)$ is true.

2^{nd} step. Let $P(K)$ is true

$P(K): (AB)^K = A^K . B^K$ ——— (1)

Now we have to prove that $P(K+1)$ is also true.

$\boxed{P(K+1): (AB)^{K+1} = A^{K+1} . B^{K+1}}$ ☆

$LHS = (AB)^{K+1}$

$= (AB)^K . (AB)^1$

↓ By equation (1)

$= A^K . B^K . AB$ $\quad (\because AB^n = B^n A)$

$AB^K = B^K . A$

$= A^K . AB^K . B$

$= A^{K+1} . B^{K+1}$

$= RHS$

$\therefore P(K+1)$ is also true.

$\therefore P(n): (AB)^n = A^n . B^n$ is also true.

MISCELLANEOUS EXERCISE ON CHAPTER $\boxed{3}$

$\boxed{\text{Q13, 14, 15}}$

Q.13

$A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ $\quad A^2 = I$

I = identity matrix

2×2

$\Rightarrow A^2 = I$

$\Rightarrow A \cdot A = I$

$\Rightarrow \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} \cdot \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\rightarrow \cdot \downarrow$

$\Rightarrow \begin{bmatrix} \alpha^2 + \beta\gamma & \cancel{\alpha\beta} - \cancel{\alpha\beta} \\ \cancel{\gamma\alpha} - \cancel{\alpha\gamma} & \gamma\beta + \alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \alpha^2 + \beta\gamma \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

comparison

$\alpha^2 + \beta\gamma = 1 \quad \Rightarrow \quad \boxed{0 = 1 - \alpha^2 - \beta\gamma}$

Q. 14

$A^T = A$	Sym ✓	Sym ✗	Sym ✗	Sym ✓
$A^T = -A$	Skew ✗	Skew ✓	Skew ✗	Skew ✓

$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ = Zero matrix = (O)

$\boxed{Q.15}$ $A \rightarrow$ square matrix

$\underbrace{A^2 = A}$ ✓

$$\begin{aligned} A^3 &= A^2 \cdot A \\ &= A \cdot A \\ &= A^2 \\ &= A \end{aligned}$$

$(I+A)^3 - 7A = ?$

?

$I \cdot A = A \cdot I = A$

$I^2 = I^3 = I^n = I$

$(a+b)^3 = a^3 + b^3 + 3ab^2 + 3a^2b$

$(I+A)^3 - 7A$

$= I^3 + A^3 + 3IA^2 + 3I^2A - 7A$

$= I + A + 3A^2 + 3A - 7A$

$= I + \underbrace{A + 3A + \cancel{3A}}_{7A} - \cancel{7A}$

$= I$

Option (C)